Sample Questions

Calculus III

1. Given that $f(x) > 0$ and $g(x) > 0$, and $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$, and if we know that $\int_1^\infty g(x)dx$ converges, then what can we say about the convergence or divergence of $\int_1^\infty f(x)dx$?

2. Integrate the following:

   (a) $\int \sinh(\ln x)dx$

   (b) $\int \frac{x^4}{1+x}dx$

   (c) $\int \frac{2x}{1+x^4}dx$

   (d) $\int x\cos^{-1}(x^2+1)dx$

   (e) $\int \frac{\sec^2\sqrt{2x}}{\sqrt{x}}dx$

   (f) $\int \sin(\ln x)dx$

3. Determine if the following improper integrals converge or diverge:

   (a) $\int_0^\infty \frac{|\sin x|+1}{e^x}dx$

   (b) $\int_{-\infty}^0 e^{-|x|}dx$

   (c) $\int_1^\infty e^x dx$

   (d) $\int_{-\infty}^\infty \frac{x^3}{e^x+e^{-x}}dx$

   (e) $\int_2^\infty \frac{1}{x^{0.5}\ln x}dx$

   (f) $\int_1^\infty \frac{((\ln t)^{-2}}{t^2}dt$

   (g) $\int_{-\infty}^\infty \frac{1}{e^x(x^4+1)}dx$

   (h) $\int_0^1 \frac{1}{\sqrt{1-x^2}}dx$